

# The Stimulated Dielectric Wake-Field Accelerator: A Structure with Novel Properties

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**Abstract.** We describe a multi-bunch wake-field accelerator in an annular, cylindrical dielectric structure in which many modes can participate in the wake-field formation, and where the wake-field period equals the period of a train of drive bunches of electrons. The structure can be designed so that the  $TM_{0n}$  modes, all with phase velocities equal to the bunch velocity, have nearly equally-spaced eigenfrequencies, and thus interfere constructively. The composite wake-field is shown to cause highly peaked axial electric fields localized on each driving bunch in a periodic sequence of bunches. This permits stimulated emission of wake-field energy to occur at a rate that is larger than the coherent spontaneous emission from a single driving bunch having the same total charge. We present calculations for an annular alumina structure which will use 2 nC bunches of electrons obtained from a 100MeV rf linac operating at 11.4Ghz. Numerical examples are given, including acceleration of a test bunch to >200 MeV in a structure 75 cm long, using three drive bunches.

## INTRODUCTION

In the conventional dielectric wakefield accelerator, a dielectric-lined waveguide supports wake fields with longitudinal electric fields induced by the passage of an electron bunch of high charge number (the “driving bunch”). Phase velocities for the modes of dielectric-lined waveguide can be less than the speed of light [1], so that Cerenkov radiation occurs[2], manifesting itself as a wake-field that fills the waveguide behind the driving bunch. If a “test bunch” of low charge number is injected at a suitable interval after the driving bunch, it can move in synchronism with the wake fields and experience net acceleration[3-5]. This approach for development of novel accelerators is appealing because no external source of rf power is required for acceleration, and because high gradient longitudinal fields are predicted for achievable high intensity driving bunches. For example, Rosing and Gai[4] consider a 100 nC, 1 mm long driving bunch passing through a dielectric lined waveguide with inner radius of 2mm and outer radius of 5mm; they took the relative dielectric constant of the outer liner to be 3.0. For this they predict a peak wake field accelerating gradient of 240 MV/m, a value about 14 times that at the Stanford Linear Collider. Experimental confirmation of wake-field generation in a dielectric-lined waveguide has been obtained[5] using 21 MeV driving bunches of 2.0 - 2.6 nC and 15 MeV test bunches of much lower charge. Acceleration gradients of 0.3-0.5 MV/m were observed in the experiments, in agreement with supporting theory. Acceleration gradients in all dielectric-lined waveguides must be below the breakdown field of the dielectric[6]. This will limit achievable gradients in any dielectric lined waveguide to a level that may not be as high as 240 MV/m.

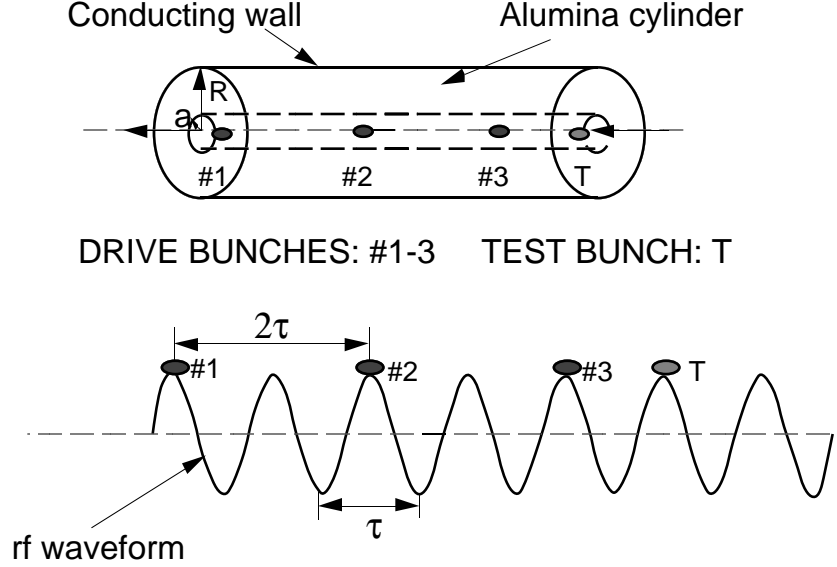
In this, we describe a different type of dielectric wake-field accelerator, in which the superposition of wake-fields from a train of drive bunches provides the field that accelerates a trailing test bunch. Imagine a waveguide lined with a dielectric shell, along which axis are injected high energy (~100 MeV) short (~ 1 psec) electron current pulses

obtained from an rf linac. If the dielectric constant and geometrical dimensions are adjusted correctly, then the waveguide modes that travel at the axial speed of the particles can have phase velocities less than the speed of light, so that Cerenkov emission can occur; the emission manifests itself as a periodic wake-field that fills the waveguide downstream from these “driving bunches”. Because of the radiative loss of energy, the drive bunches slow down. If a “test” bunch of low charge number but the same energy is injected at a suitable interval behind the driving bunches, it will move synchronously with the wake-fields and can experience net acceleration. This approach for the development of new electron or positron accelerators is appealing because no external source of rf power is required (beyond the conventional rf source to drive the injector), and because high-gradient fields may be obtained. The dielectric wake-field principle receiving greatest attention[4] uses a single high charge drive bunch which is expected to create a strong accelerating wake on its own. But, difficulties attend the creation, propagation and emittance control for such a high charge bunch; so it may be more advantageous to obtain the same accelerating gradient using a train of smaller charge bunches that should be easier to control.

The particular approach presented here enjoys two uncommon virtues. The first arises because many waveguide modes can participate in wakefield formation. These modes are designed so that, with phase velocities equal to the bunch velocity and to one another, the modes have nearly equally-spaced eigenfrequencies. This leads to a strongly-peaked spatiotemporal superposition of many co-propagating waveguide modes, so the net wake-field amplitude can be significantly larger than the amplitudes of individual modes. The second virtue arises because the near-periodic character of the wake-fields allows constructive interference of field amplitudes from successive bunches. Thus one adds to the *spontaneous* Cerenkov wake-field emission of the first drive bunch that experiences no rf fields, the *stimulated* Cerenkov emission from a train of following drive bunches that experience the wakes of preceding bunches. (The terms *spontaneous* and *stimulated* are used here to distinguish between radiation in the absence or presence of an ambient radiation field.) The bunches are assumed to be identical, and each bunch is injected to move initially with near-synchronism in the net wake-field of prior bunches. It will be shown that stimulated emission from each trailing bunch can exceed the spontaneous Cerenkov emission from a single bunch moving alone. Consequently, the drag field that decelerates a “dressed” bunch can be much larger than the drag field acting on a “bare” bunch (terms in quotes refer to the presence or absence of decelerating wake-fields from prior bunches). Thus a dressed bunch leaves behind a stronger wake than a bare bunch; and so forth for succeeding dressed bunches. Of course, successive wake maxima are not exactly periodic, and decelerating electrons can slip behind the wake-field maxima; so the cumulative superposition of wakes is less than a sum of peak values. Nevertheless, we find that a stronger wake-field might be produced by a multi-bunch train, than by a single bunch containing the total charge of all the particles. Detailed calculations of the energy loss of the drive bunches and the acceleration of the test bunch are dealt with in a paper [7] in Physical Review E. In that paper the configuration is two-dimensional rectangular dielectric slabs and a sheet beam, whereas we describe here the interaction in more practical cylindrical [8] geometry.

The wake-field concept described here should lend itself to staging, thus allowing acceleration to high energies. In the simplest staged configuration, drive bunches can be injected into a section of straight dielectric waveguide, with the spent drive bunches exiting along a collinear path. Then the waveguide modes carrying the

wake fields can be diffracted around a gentle S-bend in the waveguide and coupled to a second straight section parallel to the first, wherein the test particles are synchronized to enter in the accelerating phase. This non-collinear arrangement permits the addition of energy to sustain the accelerating wake-field, prevents the spent bunches from slipping back and re-absorbing their own wake field energy, and comprises one stage of a much larger system.



**FIGURE 1.** Schematic of the wake field element(above) and bunch timing(below).

## THEORY

The model analyzed here is a cylindrical waveguide, consisting of dielectric material with an axisymmetric hole; the outer surface of the cylinder is coated with a low-loss conductor. The dielectric constant  $\kappa$  is assumed to be independent of frequency. The geometry is depicted in Fig. 1. Electrons are injected along the  $z$ -axis in discrete axi-symmetric bunches, with charge density given by  $\rho(r, z, t) = -Ne(1/2\pi r)\delta(r)f(z - vt)$ , where  $e$  is the magnitude of the electron charge;  $N$  is the total charge number in the bunch;  $\delta(r)$  is the transverse charge distribution, assumed to be of infinitesimal width in  $r$ ; and  $f(z - vt)$  is the longitudinal charge distribution for bunch particles moving at axial speed  $v$ . For this geometry, orthonormal wave functions can be found for the electromagnetic fields that separate into  $TE$  and  $TM$  classes. Only the  $TM$ -modes have an axial electric field; these are the modes considered here. For the geometry shown in Fig. 1, conditions can be found where all  $TM$  modes have phase velocities equal to  $v$ , corresponding to wake-fields that move in synchronism with the electron bunches. The field components for the complete orthonormal  $TM$  mode set are given by

$$E_z(r, z, t) = \sum_{m=0}^{\infty} E_m \frac{f_m(r)}{\alpha_m} e^{-i\omega_m z_0/v}, \quad (1)$$

where

$$f_m(r) = \frac{1}{P_0(k_{2m}, R, a)} \begin{cases} P_0(k_{2m}, R, a) I_0(k_{1m}r), & 0 \leq r \leq a \\ P_0(k_{2m}, R, r) I_0(k_{1m}a), & a \leq r \leq R \end{cases}, \quad (2)$$

taking  $z_o = z - vt$ .

As we will see later, the field amplitude  $E_m$  is expressed by the product of the Coulomb field and a structure factor that depends on the electron bunch size. In the above equations,

$P_0(k, R, r) = J_0(kR)N_0(kr) - J_0(kr)N_0(kR)$  and  $P_1(k, R, r) = J_0(kR)N_1(kr) - J_1(kr)N_0(kR)$ ;  $J_m(x)$  and  $N_m(x)$  are  $m$ -th order Bessel functions of the first and second kinds, and  $I_m(x)$  is the modified Bessel function;  $a$  and  $R$  are radii of the central vacuum hole and the outer waveguide wall, respectively. The normalizing constant is found to be

$$\alpha_m = \frac{1}{2} \left\{ \frac{1}{\kappa} \left( \frac{\gamma}{\gamma_\kappa} \right)^2 I_1^2(k_{1m}a) \left[ \left( \frac{R}{a} \right)^2 \left( \frac{P_1(k_{2m}, R, R)}{P_1(k_{2m}, R, a)} \right)^2 - 1 \right] - (\kappa - 1) I_0^2(k_{1m}a) - I_1^2(k_{1m}a) \right\};$$

where  $\beta = v/c$ ;  $\gamma = (1 - \beta^2)^{-1/2}$ ; and  $\gamma_\kappa = (\kappa\beta^2 - 1)^{-1/2}$ . The (evanescent) transverse wave number in the vacuum is  $k_{1m}$ ; the (real) transverse wave number in the dielectric is  $k_{2m}$ , and  $k_{1m} = \omega_m/c\beta\gamma = k_{2m}\gamma_\kappa/\gamma$ . The eigenfrequencies are  $\omega_m = c\beta\gamma k_{1m} = c\beta\gamma_\kappa k_{2m}$ . Since all  $TM$  modes have phase velocities equal to  $v$ , we also have  $\omega_m = c\beta k_{z,m}$ . For the fields, ortho-normalization obtains in the form

$$\int_0^R dr r E_{z,m} D_{z,n}^* = \delta_{mn} \frac{a^2}{\sqrt{\alpha_m \alpha_n}} \epsilon_o E_m E_n \exp[-iz_o(\omega_m - \omega_n)/v], \quad (3)$$

where  $D_{z,n}^* = \epsilon E_{z,n}^* = \kappa \epsilon_o E_{z,n}^*$  in the dielectric, and  $D_{z,n}^* = \epsilon_o E_{z,n}^*$  in the vacuum hole. The dispersion relation is found to be

$$\frac{I_1(k_1 a)}{I_0(k_1 a)} = \frac{\kappa k_1}{k_2} \frac{J_0(k_2 R) N_1(k_2 a) - J_1(k_2 a) N_0(k_2 R)}{J_0(k_2 R) N_0(k_2 a) - J_0(k_2 a) N_0(k_2 R)}. \quad (4)$$

It is noted that one can have eigenfrequencies with nearly periodic spacing, since  $k_{2m}(R - a) \rightarrow (n + 1/2)\pi$  as  $\kappa \rightarrow \infty$ . As  $m \rightarrow \infty$  the asymptotic eigenfrequency spacing approaches  $\Delta\omega = \pi c \beta \left[ (R - a) \sqrt{\kappa\beta^2 - 1} \right]^{-1}$ . The wake-field is more strongly peaked and more closely periodic in  $z_o$  as the eigenfrequencies become more nearly periodic, i.e. as a higher value of  $\kappa$  is used.

To find wake-fields induced by an electron bunch, one expands in orthonormal modes the solution of the inhomogeneous wave equation,

$$\left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{\partial^2}{\partial z^2} - \frac{\kappa(r)}{c^2} \frac{\partial^2}{\partial t^2} \right] E_z(r, z, t) = S_z(r, z, t), \quad (5)$$

with the source function  $S_z(r, z, t) = \mu_o \frac{\partial j_z}{\partial t} + \frac{1}{\epsilon_o} \frac{\partial \rho}{\partial z}$ , where the  $z$ -component of the current density is  $j_z(r, z, t) = v\rho(r, z, t)$ , and where  $S_z(r, z, t) = 0$  for  $r \geq a$ . A complete solution can be constructed from the fields, since these are solutions of Eq. 5 with  $S_z(r, z, t) = 0$  everywhere. We expand the solution of Eq. 5 in the interval  $0 \leq r \leq R$  in a Fourier integral.

$$E_z(r, z, t) = \sum_{m=0}^{\infty} \frac{1}{\alpha_m} f_m(r) \int_{-\infty}^{\infty} dk A_m(k) e^{-ikz_o}. \quad (6)$$

Inserting Eq. 6 into Eq. 5, and multiplying both sides by  $w(r)D_{z,n}^*(r, z, t)$  gives

$$A_m(k) = \frac{1}{2\pi\alpha_m(k^2 - \omega_m^2/v^2)} \int_0^R r' dr' \int_{-\infty}^{\infty} dz'_o S_z(r', z'_o) \kappa(r') f_m(r') w(r') e^{ikz'_o} \quad (7)$$

where the weighting factor is  $w(r) = [-\gamma^2, \gamma_\kappa^2]$  in the intervals  $[(0 \leq r \leq a), (a \leq r \leq R)]$ , respectively. Then, integrating over  $k$ , with due regard for choice of the contour of integration consistent with causality, one obtains the Green's function, with the aid of Eq. 3, and for a Gaussian bunch:

$$\rho(r, z, t) = -\frac{Ne\delta(r)}{2\pi r\Delta z} \exp\left[-(z_o/\Delta z)^2\right], \text{ one finds}$$

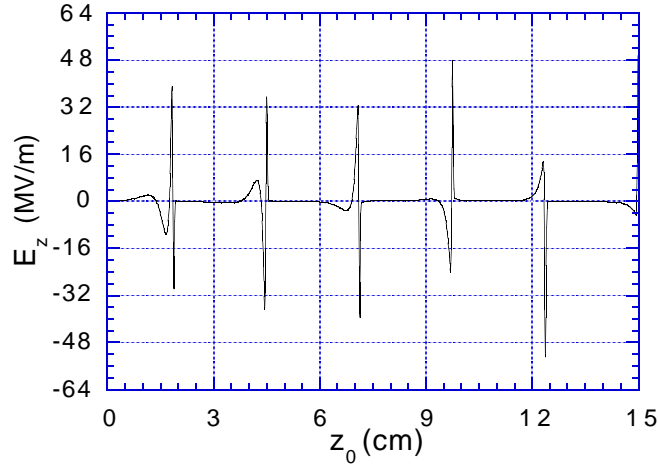
$$E_z(r, z, t) = -E_o \sum_{m=0}^{\infty} \frac{f_m(r)}{\alpha_m} e^{-(\omega_m\Delta z/2v)^2} e^{-i\omega_m z_o/v}. \quad (8)$$

In Eq. 8,  $E_o = -Ne/4\pi\epsilon_o a^2$  is the Coulomb field of the bunch at the edge of the hole, and causality dictates that the results are valid only behind the bunch, i.e. for  $z_o \leq 0$ ; ahead of the bunch the fields are of course zero.

Eq. 4 has been evaluated for a waveguide with  $a = 0.375$  mm,  $R = 0.488$  cm, and  $\kappa = 9.43$ . It is assumed that  $\kappa$  is independent of frequency. The phase velocity of all the  $TM$  modes corresponds to the speed of an electron bunch with  $\gamma = 200$ , i.e.  $0.999988c$ . For this case, the first frequency interval  $(\omega_2 - \omega_1)/2\pi = 10.797$  GHz, while the asymptotic interval  $\Delta\omega/2\pi$  is 11.410 GHz. It is thus seen that conditions can be found where the eigenmodes all have equal phase velocities with nearly equal eigenfrequency spacings. This sets the fundamental period of the wake-field structure at

5.2 cm. A suitable accelerator that would power this wake-field element would use an X-band source with frequency of 11.4 GHz, or free space wavelength of 2.6 cm. A 100 MeV pulse of electrons about 1 psec in length would be produced whenever a photocathode rf gun is struck with a narrow pulse of UV light from a synchronized laser. The light pulse should be timed to arrive at the photocathode when the rf field has its maximum. The three drive bunches, to be described later (each separated by  $4\pi$  in rf phase), and one test particle bunch (separated by  $2\pi$ ), comprise the electron bunch sequence. One laser pulse will be split into 4 pulses, spaced so as to arrive at the cathode at the rf field maxima. Thus the drive pulses are to be spaced by 5.2 cm (the first subharmonic) and the test bunch follows the third drive bunch by 2.6 cm. Figure 1 shows the timing sequence referenced to the rf waveform.

Our theoretical results were evaluated numerically for comparison with wake-field experiments conducted at Argonne National Laboratory [5]. Good agreement was obtained for the wake-field structure, intensity, and mode frequencies, requiring only the first few modes that make significant contributions (also noted by the Argonne group). On the other hand, our design choice for cylindrical waveguide shows that the addition of many modes, as required here, results in a more sharply -defined wake-field structure: this is evident in Figure 2 where the wake-fields trailing a *single* bunch are shown. This is similar to the result we found for the planar slab example[7].



**FIGURE 2.** On-axis wake-field from a 2 nC, 100 MeV, 0.25mm long Gaussian bunch after it has traveled 15 cm left to right in the cylindrical waveguide with parameters as in Figs. 2 and 3. At  $z_0 = 15$  cm, the wake-field amplitude is 57.6 MV/m.

The choice of these dielectric-lined waveguide parameters is seen to result in a spacing of 5.2 cm between the first few sharply-peaked positive-polarity wakefield features. This spacing is equal to twice the rf period of an 11.4 GHz injector gun, or for that matter, an rf linac. Therefore, if successive bunches were injected into the dielectric waveguide, the second drive bunch will find itself riding just on the crest of the first decelerating wake feature generated by the first drive bunch. Instead of generating only a spontaneous Cerenkov wake as did the first bunch, the second bunch will be decelerated in the field of the first wake, and its energy will be radiated as additional stimulated Cerenkov energy which builds up its own wake. Successive bunches will

interact similarly. To make this quantitative, one calculates the incremental energy  $\Delta W$  radiated into the waveguide by a bunch in advancing a distance  $\Delta z$ , and equates that to the energy loss rate of the bunch. This loss rate is identified with a drag field  $E_{drag}$  acting on the bunch. Thus,  $QE_{drag} = \Delta W/\Delta z$ . For a bare bunch,  $E_{drag} = E_{spont}$ , the drag field corresponding only to spontaneous emission. But for a dressed bunch that follows behind  $N$  prior bunches, the drag field consists of the spontaneous drag field added to the combined wakefields of the preceding bunches. The total wakefield is next calculated by adding up the incremental wakefield energies from each bunch up to that location and then finding the resulting electric field, and so on. It is also noted that perfect synchronism is assumed in the above simplified discussion, so that wake amplitudes (and not energies) are added constructively. More highly relativistic bunches can maintain synchronism while losing a larger fraction of their initial energy, as compared with bunches of lesser energy, since velocity slip is lower, i.e.  $\Delta\beta \approx \Delta\gamma/\gamma^3$ .

Now we turn to considerations of energy loss by the bunch. The rate of energy accumulation with distance in wake fields behind the bunch  $dW/dz$  is given by integrating the field energy density over the waveguide cross-section, thus leading to the following relation.

$$\begin{aligned}
\frac{dW}{dz} &= \frac{1}{4} \sum_{m=0}^{\infty} \int_0^{2\pi} d\theta \int_0^R r dr \left\{ \epsilon(r) [E_{r,m}^2 + E_{z,m}^2] + \mu_o H_{\theta,m}^2 \right\} \\
&= \frac{\epsilon_o E_o^2 \pi R^2}{2} F \sum_{m=0}^{\infty} \frac{h^2(\xi_m)}{\alpha_m^2} \left\{ \frac{1}{R^2} \int_0^a I_0^2(k_{1m}r) r dr + (1 + \beta^2) \gamma^2 \frac{1}{R^2} \int_0^a I_1^2(k_{1m}r) r dr \right. \\
&\quad \left. + \kappa \left( \frac{I_0(k_{1m}a)}{P_0(k_{2m}, R, a)} \right)^2 \left[ \frac{1}{R^2} \int_a^R P_0^2(k_{2m}, R, r) r dr + (1 + \kappa\beta^2) \frac{\gamma_k^2}{R^2} \int_a^R P_0^2(k_{2m}, R, r) r dr \right] \right\} \\
&\hspace{15em} (9)
\end{aligned}$$

In Eq. 9,  $F$  is a scale factor fixed by balancing the energy loss rate between drag on the bunch, and increase in the wake-field energy, as described in the prior paragraph. For a bare bunch  $F = 1$ . This scaling procedure assumes that the source current and charge distributions remain constant during the interaction. If not, the individual mode amplitudes must be adjusted iteratively; this amounts to introduction of a  $z$ -dependent structure factor  $h(\xi_m)$ . For the Gaussian bunch,  $h(\xi_m) = \exp(-\xi_m^2)$ , with  $\xi_m = \omega_m \Delta z / 2v$ .

For the drive bunch charge of 2 nC, in Figure 2 we compute a wake-field of 57.6 MV/m and find a drag field of 11.45 MV/m (the latter is calculated by equating the spatial rate of radiation energy loss from the bunch, given by eq. 9, to the product of a “drag” electric field and the bunch charge). We note that this gives a wake-to-drag field ratio, the so-called “transformer ratio”[9] of 5.0: transformer ratios above two have been

shown to be possible for multi-mode waveguides[10] (on the other hand, the ratio we quote is based on bunch average quantities, and so may differ on that account).

In the following simple model for the buildup of a *cumulative* wake-field from a multiple-drive bunch train, the bunches are taken as point charges which remain perfectly synchronized with the wake-fields. The energy radiated into the wake-field of the  $n$ -th bunch can be written in terms of the net drag on the charge  $Q$  as

$$\frac{dW_n}{dz} = R \left[ \left( \sum_{i=1}^n E_i \right)^2 - \left( \sum_{i=1}^{n-1} E_i \right)^2 \right] = Q \left[ E_{spon} + \sum_{i=1}^{n-1} E_i \right] = Q E_{drag}, \quad (10)$$

from which the combined wake-field is found to be:

$$\sum_{i=1}^n E_i = \left[ \left( \sum_{i=1}^{n-1} E_i \right)^2 + \frac{Q}{R} \left( E_{spon} + \sum_{i=1}^{n-1} E_i \right) \right]^{1/2}. \quad (11)$$

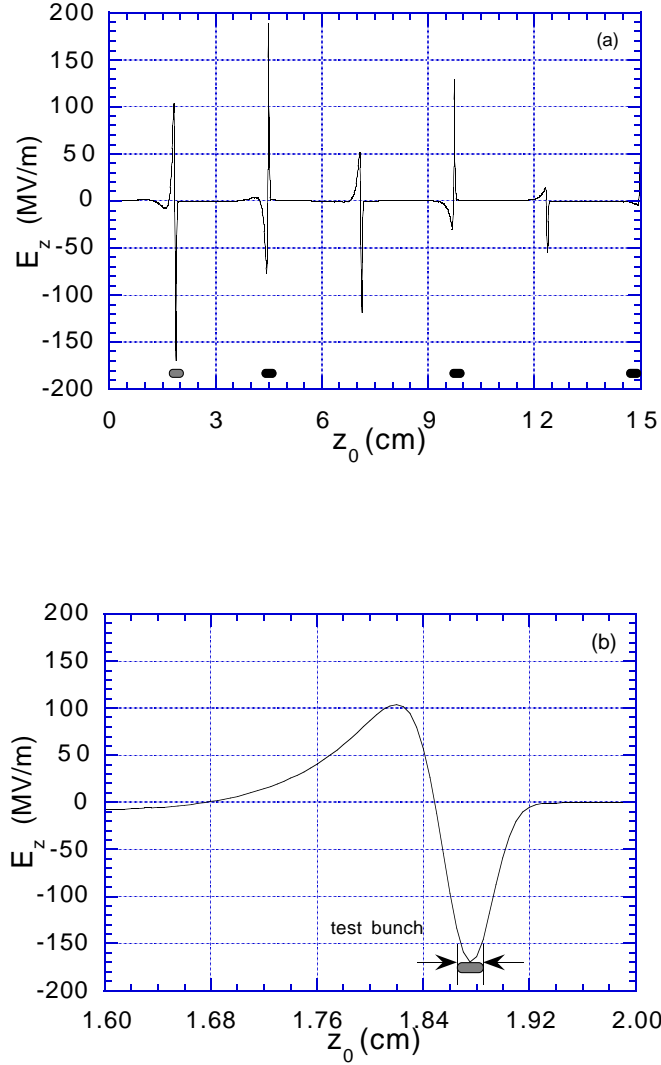
The factor “ $R$ ” in eqs. 10 and 11 is obtained from the coefficient of the electric field squared in Eq. 9(not to be confused with  $R$ , the radius). The individual wake-field from the  $i$ -th bunch is  $E_i$ . One can appreciate from Eqs.10 and 11 that participation of many co-propagating modes, and stimulated emission from a periodic sequence of driving bunches, can increase dramatically the extraction of energy from the bunches, which in turn promotes the buildup of an intense overall wake-field after passage of a relatively few moderate-charge driving bunches. These results suggest that high wake-fields can be obtained in this manner, without the use of a single, high- $Q$  bunch and its attendant difficulties.

The conceptual model discussed above assumes perfect synchronism between driving bunches and peak wake-fields, and assumes the wake-field amplitude to be uniform over the finite spatial extent of each bunch. The problem has been examined with greater accuracy in a numerical simulation, using 100 particles per bunch, and taking slip and actual wake-field amplitude variations into account. Particles in each 0.25 mm long bunch are injected around the peak of the wake-field from prior bunches. The energy loss rate from each bunch is given by Eq. 9, with successive values of  $F$  found from the drag field on that bunch, i.e. from the sum of its spontaneous drag field  $E_{spon}$  and the net wake-field from prior bunches. Particles in a given bunch obey the one-dimensional equation of motion  $d\gamma/dz = (e/mc^2) E_{drag}$ , evaluated at each particle's instantaneous location, where  $E_{drag}$  is obtained from Eq. 10. The initial energy of each bunch is chosen with  $\gamma_{initial} = 200$ , or about 100 MeV. In this model, the bunches--now distributed spatially--will lose energy and thus can slip with respect to the wake-fields. Motion in the  $x$ - $y$  transverse plane is neglected.



## RESULTS

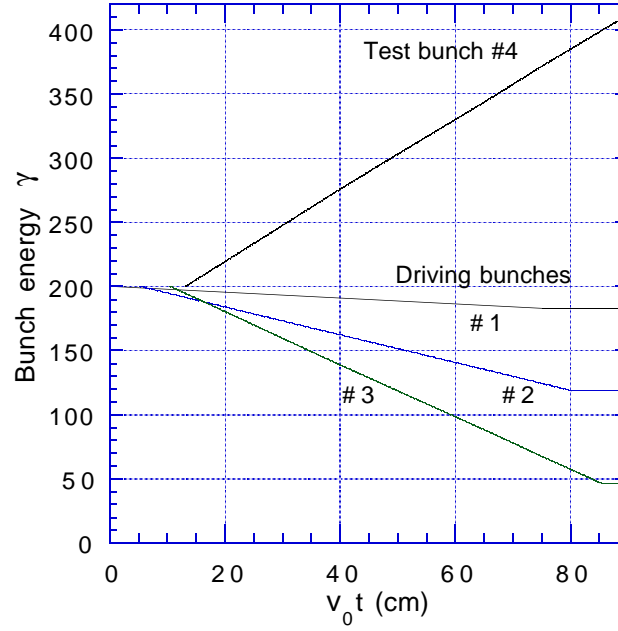
Figure 3a shows the buildup of the wake-fields in the device under study here, if three 2nC drive bunches moving in a wake-field structure are used. Superposition of wakes is seen to result in a wake-field amplitude at the third bunch of 188 MV/m, which is greater than the wake-field (173 MV/m) of a single bunch of three times the charge. However, a 6 nC, 1 psec bunch may be beyond current state of the art.



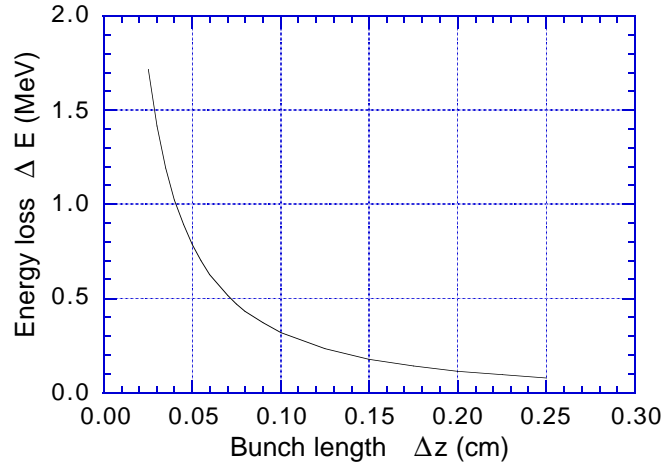
**FIGURE. 3.** (a) Cumulative wake-field set up by three identical successive drive bunches, at the time the first bunch has moved 15 cm along the waveguide. The bunch locations are indicated as black dots on the figure. (b) Location of the test bunch in the accelerating wake field behind the third drive bunch near the entrance of the accelerator, shown on an expanded scale.

Figure 4 shows the average energy increase of the test bunch (#4), about 105 MeV in 75 cm; this is less than the maximum of 170 MV/m  $\times$  0.75 m = 127 MeV because of variations in the field across the bunch, as shown in Figure 3b. Beam loading is

neglected for the test bunch. As others have concluded[10], collinear transport of drive and test bunches imposes limitations on the wake-field accelerator, which suggests that the radiation and the drive particles should be separated. Nevertheless, the acceleration field of 140 Mv/m that was obtained makes this an attractive concept to explore further.



**FIGURE. 4.** History of the energy of the three drive bunches and the test bunch, up to the time the test bunch has left the 90 cm long waveguide.



**FIGURE. 5.** Energy loss to wake-field radiation as a function of bunch length for a 2 nC, 100 MeV Gaussian bunch traversing a 15 cm long waveguide.

In Figure 5, we show calculations of the energy loss of a single 100 MeV bunch traversing a fixed length of cylindrical dielectric loaded waveguide with parameters the same as for the acceleration experiments. For a constant charge of 2 nC, the figure shows the energy loss as a function of rms axial bunch length, for a Gaussian distribution. As the energy decrease of the particles is readily monitored, and is strongly sensitive to the bunch length, it appears that there is the possibility for a simple diagnostic for bunch length. A study of this would further test our theory, and provide confidence in using it as an on-line bunch monitor (with an energy loss  $\sim 1\%$ , such a diagnostic can be considered to be passive).

Our calculations have shown that a multi-bunch wake-field accelerator may have some advantages over the single bunch version (which contains a substantially larger charge). The stimulated buildup of wake-fields allows one to produce wake-fields well in excess of 100 MeV/m, using short bunches that contain modest charge ( $\sim 2$  nC). Nevertheless, at this time there remain some problems which can be resolved only by further analysis and experimentation: among them stability, very short-pulse dielectric breakdown, and dispersion.

## ACKNOWLEDGMENT

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